RANI RASHMONI GREEN UNIVERSITY
TARAKESWAR, HOOGHLY

M. SC. SYLLABUS OF MATHEMATICS

(FOR 2 YEARS CBCS)

Subject – Mathematics					
Programme Structure					
SEM	Course Code	Course Title	Full Marks	Credit (L+T+P)	Lecture Hours
I	EVS	Environmental Science	40+10*	4	40
	MTM – 101	Abstract Algebra	40+10	4	40
	MTM – 102	Real Analysis	40+10	4	40
	MTM – 103	Complex Analysis	40+10	4	40
	MTM – 104	Topology	40+10	4	40
	MTM – 105	General Mechanics	40+10	4	40
TOTAL			300	24	240
II	MTM – 201	CBCS – 1	40+10	4	40
	MTM – 202	Linear Algebra	40+10	4	40
	MTM – 203	Functional Analysis	40+10	4	40
	MTM – 204	Ordinary Differential Equation and Special Functions	40+10	4	40
	MTM – 205	Algebraic Topology / Continuum Mechanics	40+10	4	40
	MTM – 206	Calculus on R^n and Elements of Operations Research (OR)	40+10	4	40
TOTAL			300	24	240
III	MTM – 301	CBCS – II	40+10	4	40
	MTM – 302	Numerical Analysis (Theory (24 Marks) and Practical (16 Marks) in C/R/MATLAB)	40+10	4	40
	MTM – 303	Partial Differential Equation (PDE)	40+10	4	40
	MTM – 304	Differential Geometry and Manifold Theory	40+10	4	40
	MTM – 305	Integral Equations and Integral Transforms	40+10	4	40
	MTM – 306	Special Paper-1	40+10	4	40
TOTAL			300	24	240
IV	MTM – 401	Probability and Stochastic Process	40+10	4	40
	MTM – 402	Nonlinear Differential Equation and Dynamical System / Number Theory	40+10	4	40
	MTM – 403	Discrete Mathematics	40+10	4	40
	MTM – 404	Special Paper-2	40+10	4	40
	MTM – 405	Project	50	4	80
	MTM – 406	Internship	50	4	80
TOTAL			300	24	320
GRAND TOTAL			1200	96	1040

Theory – 50 Marks, Written - 40 Marks, Internal Assessment – 10 Marks.
For Practical / Project, lecture hours would be twice of the theory.
* Each student will obtain marks based on the plantation and growing up of a sampling that would produce fruits and attract the birds / animals.

List of Special Papers (MTM – 306):

1. **Advanced Algebra-I**
2. **Advanced Complex Analysis-I**
3. **Advanced Differential Geometry-I**
4. **Advanced Functional Analysis-I**
5. **Advanced Real Analysis-I**
6. **Advanced Topology-I**
7. **Fluid Mechanics-I**
8. **Harmonic Analysis-I**
9. **Mathematical Biology-I**
10. **Operations Research-I**
11. **Plasma Dynamics-I**
12. **Solid Mechanics-I**

List of Special Papers (MTM – 404):

1. **Advanced Algebra-II**
2. **Advanced Complex Analysis-II**
3. **Advanced Differential Geometry-II**
4. **Advanced Functional Analysis-II**
5. **Advanced Real Analysis-II**
6. **Advanced Topology-II**
7. **Fluid Mechanics-II**
8. **Harmonic Analysis-II**
9. **Mathematical Biology-II**
10. **Operations Research-II**
11. **Plasma Dynamics-II**
12. **Solid Mechanics-II**

SEMESTER-I
Course Code: MTM-101
Abstract Algebra: 50 Marks (4 CP)

Groups: Direct product and semi-direct product of groups. Automorphisms and inner automorphisms. Conjugation, conjugacy relation, conjugacy class equation and consequences, Group actions, applications of group actions, Burnside counting principle, generalized Cayley's theorem. Simplegroups, p-groups, Cauchy's theorem, Converse of Lagrange's theorem for finite abelian groups, Sylow's theorems and some applications, classifications of finite groups. Normal and subnormal series, composition series, solvable groups and nilpotent groups, Jordan-Hölder theorem and its applications. Finitely generated abelian groups, free abelian groups.

Rings: Prime and maximal ideals, field of quotient of an integral domain, polynomial rings, divisibility theory in integral domain, Euclidean Domain, Principal Ideal Domain, irreducible and prime element, Unique Factorization Domain, Gauss' Theorem, Eisenstein's irreducibility criterion.

Fields: Field extension, Algebraic and transcendental extensions, finite extension, algebraically closed field, splitting field, separable extension, impossibility of some constructions by straightedge and compass, finite fields and their properties, normal extension, Galois extension, Galois group, Galois theory, solution of polynomial equations by radicals, insolvability of the general equation of degree 5(or more) by radicals.

References:
1. David S. Dummit, Richard M. Foote, Abstract Algebra, 3rd edition, John Wiley & Sons, 2003.
2. J. B. Fraleigh, A first course in Abstract algebra, 7th edition, Addison-Wesley, 2003.
3. Joseph A Gallian, Contemporary Abstract Algebra, Cengage Learning India Pvt. Ltd., 2019.
4. T. W. Hungerford, An Introduction to Abstract Algebra, 3rd edition, Brooks/Cole Cengage Learning.
5. D. S. Malik, J. N. Mordeson and M. K. Sen, Fundamentals of Abstract Algebra, McGraw Hill.

SEMESTER-I
Course Code: MTM-102
Real Analysis: 50 Marks (4 CP)

Functions of bounded variation**:** Definition and basic properties, Lipschitz condition, Jordan decomposition, Nature of points of discontinuity, Nature of points of non-differentiability, positive and negative variation and their properties.

Absolutely continuous functions**:** Definition and basic properties, Deduction of the class of all absolutely continuous functions as a proper subclass of all functions of bounded variation.

Lebesgue Measure: Outer Lebesgue Measure m^* in the Euclidean line and its Properties. Outer measure $\boldsymbol{\mu}^*$on S, where S is a space, the concept of **μ**-measurable sets with the help of $\boldsymbol{\mu}^*$.Necessary and sufficient condition for **μ**-measurability. Properties of **μ**-measurable sets.The structure of **μ**-measurable sets, the concept of σ-algebra, the σ-algebra of Lebesgue measurable sets.

Properties of Lebesgue measure, Vitali's theorem: The existence of a non-measurable set in the Euclidean line. The Borel sets &Lebesgue measurable sets- a comparison.

μ-measurable functions, their properties: Characteristic functions, Simple functions. Theorem relating to the non negative**μ**-measurable function as a limit of a monotonically increasing sequence of non negative simple **μ**-measurable functions.

LebesgueIntegration: Integration for simple functions and for Extended real valued **μ**-measurable functions, The countable additivity of the set of function ν_f on **M** defined by $\nu_f(M) = \int_M f$, for each set M $\in$**M** , the **σ**- algebra of **μ**-measurable sets, for a nonnegative **μ**-measurable function f.Lebesgue's monotone convergence theorem and its applications,Fatou's lemma, Lebesgue's dominated convergence Theorem. Necessary & Sufficient condition of Riemann integrability via measure; interrelation between the two modes of integration.

References:

1. P. R. Halmos, Measure Theory, Von Nostrand, New York,1950.
2. E. Hewitt & K. Stromberg, Real and abstract Analysis, Third edition, Springer.
3. G.D. Barra, Measure Theory & Integration, Wiley Eastern Limited, 1987.
Verlag,Heidelber& New York, 1975.
4. W. Rudin, Real and Complex Analysis, Tata McGraw- Hill, New York, 1987.
5. I. K. Rana, An introduction to Measure & Integration, Narosa Publishing House, 1997.
6. H. L.Royden, Real Analysis, Macmillan Pub. Co. Inc, New York, 1993.
7. J. F. Randolf, Basic Real and Abstract Analysis, Academic Press, New York, 1968.
8. C. D. Aliprantis and Owen Burkinshaw, Principles of Real Analysis, Academic Press, 2000.
9. T. M. Apostol, Mathematical Analysis, Narosa Publishing House, 1985.

SEMESTER-I
Course Code: MTM-103
COMPLEX ANALYSIS: 50 Marks (4 CP)

The Real Number System, Graphical Representation of Real Numbers, The Complex Number System, Fundamental Operations with Complex Numbers, Graphical Representation of Complex Numbers, Topological Characteristic of complex number system by considering it as a metric space, Point sets, Curves (smooth curves, piecewise smooth curve, Jordan Curve, open arc, Closed Path), regions (Simply Connected Regions, Multiply Connected Regions) and nests.

Complex valued function defined on a closed and bounded interval, Complex valued function defined on an open region of C, Limits, Theorems on Limits, Continuity, Theorems on Continuity, Sequences, Limit of a Sequence, Theorems on Limits of Sequences.

Derivative Complex valued functions, Rules for Differentiation, Derivatives of Elementary Functions, Cauchy–Riemann Equations, CR theorem, Converse of CR theorem with slight modification, holomorphic (Analytic) function, Harmonic Functions and related theorem

Complex Line Integrals, Connection Between Real and Complex Line Integrals, Contour Integrals, Cauchy's Theorem, The Cauchy–Goursat Theorem, Winding number, Cauchy's integral formula, Cauchy's integral formula for derivatives, Morera's Theorem, Cauchy's inequality, Liouville's theorem, Fundamental theorem of Algebra.

Sequences of Functions, Series of Functions, Absolute Convergence, Uniform Convergence of Sequences and Series, Power Series including some Important Theorems on Sequences of Functions, Series of Functions and Power Series.

Taylor's Theorem and Analytic Continuation, zero of an analytic function, theorems on the limit points of the set of all zeros of an analytic function, Laurent's Theorem, singularity, Classification of Singularities - isolated singularity, pole, removable singularity, essential singularity, Relation between zero and pole of a function, Entire Functions, Meromorphic Functions, Maximum modulus principle

Residues, Cauchy's Residue Theorem, Rouches's Theorem, Special Theorems Used in Evaluating Integrals, Evaluation of Definite Integrals, Evaluation of Contour Integrals

Conformal Mappings: Transformation from one Complex plane to another, Jacobian of a Transformation, The Linear Transformation, The Bilinear or Fractional Transformation, Mapping of a Half Plane onto a Circle, Some Special Transformations.

References:

1. Amritava Gupta, Principles of Complex Analysis, Academic Publishers, Kolkata, 2010.
2. Conway, J.B., Functions of one complex variable, Second Edition, Narosa Publishing House.
3. Amritava Gupta, Principles of Complex Analysis, Academic Publishers, Kolkata, 2010.
4. Sarason, D., Complex Function Theory, Hindustan Book Agency, Delhi, 1994.
5. J.W. Brown and R.V. Churchill, Complex variables and applications, McGraw Hill, 1948.
6. Ahlfors, L.V., Complex Analysis, McGraw-Hill, 1979.
7. Rudin, W., Real and Complex Analysis, McGraw-Hill Book Co., 1966.
8. Hille, E., Analytic Function Theory (2 vols.), Gonn& Co., 1959.
9. Titchmarsh, E.C., The Theory of Functions, Oxford University Press, London.
10. Ponnusamy, S., Foundations of Complex Analysis, Narosa Publishing House, 1997.

SEMESTER-I
Course Code: MTM-104
Topology: 50 Marks (4 CP)

Topological spaces, examples of topological spaces, bases and sub-bases, open and closed sets. Closure and interior – their properties and relations, exterior, boundary, accumulation points, derived sets, adherent point, dense set, G_δ and F_σ sets. Kuratowski closure operator, neighbourhood systems. Subspace or relative topology and metric topology.

Continuous, open, closed maps and their examples and characterizations, pasting lemma, homeomorphism and topological properties, hereditary property.

1st and 2nd countability axioms, separability, Lindeloffness and their relationships, characterizations of accumulation points in a 1st countable space w.r.t. sequences and Heinei's continuity criterion.

T_i spaces (i = 0, 1, 2, 3, 3½, 4, 5), their examples and hierarchy, characterizations and basic properties, Urysohn's lemma and Tieze's extension theorem (statement only).

Connected, disconnected spaces and their examples, connectedness on the real line, basic properties of connected spaces and components.

Compactness, its basic properties and characterizations, Alexander's sub-base theorem (statement only), continuous functions and compact sets, compactness and separation axioms.

Product topology of finite number of topological spaces, projection maps, product theorems of finite number of (i) Hausdorff spaces, (ii) connected spaces and (iii) compact spaces.

References:

1. K. D. Joshi, Introduction to General topology, Wiley Eastern Ltd.
2. J. Munkres, Topology, A first course, Prentice Hall, India.
3. C. Kosniwski, A first Course in Algebraic Topology, Cambridge University Press.
4. S. Willard, General Topology, Addison-Wesley.
5. J. Dugundji , Topology, Allyn and Bacon.
6. J. L. Kelley, General Topology, Van Nostrand.
7. G. F. Simmons, Introduction to topology and modern analysis, McGraw Hill.
8. Martin D. Crossley, Essential topology, Springer- Verlag.
9. Engelking, General Topology, Polish Scientific Publishers, Warszawa.
10. L. Steen and J. Seebach, Counter examples in Topology.

SEMESTER-I
Course Code: MTM-105
General Mechanics: 50 Marks (4 CP)

Mechanics of a system of particles, Constraints and their classifications, Degrees of freedom, Generalized coordinates, Virtual displacement and principle of virtual work, D'Alembert's principle, Lagrange's equations of first and second kind, Generalized forces, Uniqueness of solutions, Energy equation for conservative fields, Eulerian angles, Euler's dynamical equations.

Hamilton's variables, Hamilton canonical equations, Homogeneity of space and time, Different conservation principles, Cyclic coordinates, Routhian function, Liouville's form of a dynamical system.

Some techniques of the calculus of variations, Derivation of Lagrange's Equations from Variational Principles, Derivation of Hamilton canonical equations from Variational Principles, Hamilton's principle, Derivation of Lagrange's equations from Hamilton's principle, Derivation of Hamilton canonical equations from Hamilton's principle, Principle of least action.

Poisson's Bracket, Basic properties of Poisson's Bracket, Poisson's identity, Jacobi-Poisson Theorem, Canonical coordinates and Canonical transformations, Poincaré theorem, Lagrange's brackets and its properties, Hamilton's equations of motion in terms of Poisson's bracket, Jacobi's identity, Hamilton-Jacobi equation.

Small Oscillations, General case of coupled oscillations, Eigen vectors and Eigen frequencies, Orthogonality of Eigen vectors, Normal coordinates, Two-body problem.

References:

1. Principle of Mechanics, J. L. Synge and B. A. Griffith, 3rd Edition, McGRAW HILL, 1984.
2. H. Goldstein, Classical Mechanics, Narosa Publ., New Delhi, 1998.
3. T.W. B. Kibble and F.H. Berkshire, Classical Mechanics, 4th ed., Addison-WesleyLongman, 1996.
4. E. C. G. Sudarshan and N. Mukunda, Classical Dynamics: A Modern Perspectives,John Wiley & Sons, 1974.
5. L. D. Landau and E. M. Lifshitz, Mechanics, 3rd ed., Pergamon Press, 1982.
6. V. I. Arnold, Mathematical Methods of Classical Mechanics, 2nd ed., Springer Verlag, 1997.
7. J.R. Taylor, Classical Mechanics, University Science Books, California, 2005.
8. E.T. Whittaker, A Treatise of Analytical Dynamics of Particles and Rigid Bodies,Cambridge Univ. Press, Cambridge, 1977.
9. A. S. Ramsey, Dynamics Part-II, The English Language Book Society and Cambridge University Press.
10. S. L. Loney, An Elementary Treatise on the Dynamics Of a Particle and Rigid Bodies, Cambridge Univ. Press.
11. N.C. Rana and P.S. Joag, Classical Mechanics, Tata McGraw Hill, New Delhi,2002.
12. Analytical Mechanics, Louis N. Hand and Janet D. Finch.
13. F. Gantmacher, Lectures in Analytical Mechanics, Mir Publ., 1975.
14. N.G. Chetaev, Theoretical Mechanics, Springer-Verlag, 1990.
15. M. Calkin, Lagrangian and Hamiltonian Mechanics, World Sci. Publ., Singapore,1996.

SEMESTER-II
Course Code: MTM-201
CBCS – 1: 50 Marks (4 CP)

Matrix, Determinant of square matrix, Singular and nonsingular matrix, Inverse of a square matrix, Rank of a matrix, Solutions of a system of linear equations, Eigen values, Eigen vectors, Characteristic equation, Caley-Hamilton Theorem (Statement only), Applications of Caley-Hamilton Theorem.

Differential equations of first order and first degree – equation solvable by separation of variables, homogeneous equation, exact equation, integrating factor, linear equation, equations reducible to linear and Bernoulli's equation.

Second order linear differential equations with constant coefficient – homogeneous equation, trial solution, auxiliary equation, general solution, determination of particular integral for non-homogeneous simple problem like e^{ax}, where a is not the root of the auxiliary equation.

References:

1. Higher Algebra (Abstract and Linear), S. K. Mapa
2. Advanced Higher Algebra (Abstract and Linear), J. G. Chakraborty and P. R. Ghosh
3. An Introduction to Differential Equations, R. K. Ghosh and K. C. Maity
4. Differential Equations, J. G. Chakraborty and P. R. Ghosh
5. Discrete Mathematics, S. K. Chakraborty B. K. Sarkar
6. A Textbook of Discrete Mathematics, Swapan Kumar Sarkar
7. Introduction to Discrete Mathematics, M.K. Sen and B.C. Chakraborty

SEMESTER-II
Course Code: MTM-202
Linear Algebra: 50 Marks (4 CP)

Linear functional, dual space and dual basis, double dual and related results, Transpose of a linear transformation and matrix representation of the transpose of a linear transformation.

Characteristic and minimal polynomial, similarity of linear operator, invariant sub-spaces, primary decomposition theorem, diagonalizable form, triangular canonical form, nilpotent operator, invariants of a nilpotent operator, generalized eigen vectors, Jordan blocks and Jordan canonical form, rational canonical form.

Linear transformations on inner product spaces, adjoint of a linear operator, matrix representation of the adjoint, normal and self-adjoint operators, unitary and orthogonal operators and their matrices, orthogonal projections, spectral theorems and consequences.

Bilinear forms, symmetric and skew-symmetric bilinear forms, real quadratic forms, Sylvester's law of inertia, positive definiteness.

Generalized inverse of rectangular matrices, Moore-Penrose inverse, singular value decomposition of matrix / linear transformation.

References:

1. Kenneth Hoffman and Ray Kunze, Linear Algebra, Prentice-Hall International, 2010.
2. P. Lax, Linear Algebra, John Wiley & Sons, 1997.
3. H.E. Rose, Linear Algebra, Birkhauser, 2002.
4. S. Lang, Algebra, 3rd Edition, Springer (India), 2004.
5. Roger A. Horn, Charles R. Johnson., Matrix Analysis, 2nd edition, CUP, 2013.
6. Sheldon Axler, Linear Algebra Done Right, 3rd edition, Springer, 2015.
7. N. Jacobson, Basic algebra I, 2nd edition, Dover Publications, 2012.
8. N. Jacobson, Basic algebra II, 2nd edition, Dover Publications, 2012.
9. T. S. Blyth & E. F. Robertson,Further Linear Algebra, Springer.
10. S. Roman, Advanced Linear Algebra, 3rd edition, Springer, 2007.
11. J. H. Kwak and S. Hong, Linear Algebra, 2nd edition, Birkhauser, 2006.

SEMESTER-II
Course Code: MTM-203
Functional Analysis: 50 Marks (4 CP)

Metric Spaces, Hölder and Minkowski inequalities (statement only), continuity and uniform continuity, completeness, compactness, connectedness. Baire's category theorem, Banach'sfixed point theorem (statement only) and its application to solutions of certain systems of linear algebraic equations.

Real and Complex linear spaces. Normed linear space, Normed induced metric. Banach spaces, the spaces R^n, C^n, C [a, b], C_0, C, lp(n)(1≤p≤∞), lp(1≤p≤∞). Riesz's lemma. Finite dimensional normed linear spaces and subspaces, completeness, compactness criterion, Quotient space, equivalent norms and its properties.

Bounded linear operators, various expressions for its norm. Spaces of bounded linear operators and its completeness. Inverse of an operator. Linear and sublinear functionals, Hahn-Banach theorem in normed linear spaces and some of its applications.

Conjugate or Dual spaces, Examples, Separability of the Dual space. Reflexive spaces, weak and weak* convergence. Uniform boundedness principle and its applications. The Open mapping Theorem and the Closed graph Theorem.

Inner product spaces and Hilbert spaces, examples of Hilbert spaces, continuity of inner product, C-S inequality, basic results on Inner product spaces and Hilbert spaces, parallelogram law, Pythagorean law, Polarization identity, orthogonality, orthonormality, orthogonal complement. The Riesz representation theorem, Bessel's inequality. Convergence of series corresponding to orthogonal sequence, Fourier coefficient, Perseval identity. Riesz- Fischer Theorem.

References:

1. W.Rudin, Functional Analysis, Tata McGraw Hill.
2. B.V. Limaye, Functional Analysis, Second Edition, New Age International limited, Madras.
3. G. Bachman & L. Narici, Functional Analysis, Academic Press, 1966.
4. N. Dunford& J. T. Scwarz, Linear operators, Vol – I & II, Interscience, New York, 1958.
5. L. V. Kantorvich and G.P. Akilov, Functional Analysis, Pergamon Press, 1982.
6. E. Kreyszig, Introductory Functional Analysis with Applications, Wiley Eastern, 1989.
7. I. J. Madox, Elements of Functional Analysis, Universal Book Stall, 1992.
8. A. H. Siddiqui, K. Ahmed and P. Manchanda, Introduction to Functional Analysis with applications, Anshan Publishers, 2007.
9. A. E. Taylor, Functional Analysis, John Wiley and Sons, New York, 1958.

SEMESTER-II
Course Code: MTM-204
Ordinary Differential Equation and Special Functions: 50 Marks (4 CP)

Ordinary Differential Equation

First order ODE, Initial value problems, Existence theorem, Uniqueness,basic theorems. Ascoli Arzela theorem (statement only), Theorem on convergenceof solution of initial value problems, Picard Lindeloff theorem and Peano's existence theorem and their applications.

Higher order linear ODE, fundamental solutions, Wronskian. OrdinaryDifferential Equations of the Sturm-Liouville type and their properties,Application to Boundary Value Problems, Eigenvalues and Eigenfunctions, Orthogonality theorem, Expansion theorem. Green's function for Ordinary Differential Equations, Application to Boundary Value Problems.

Fundamental System of Integrals, Regular Integral, Equation ofFuchsian type, Series solution by Frobenius method.

Special Functions

Hypergeometric Functions, Series Solution near zero, one, and infinity. Integral Formula, Differentiation of Hypergeometric Function.

Legendre Functions, Generating Function, Legendre Functions of First and Second kind, Laplace Integral, Orthogonal Properties of Legendre Polynomials, Rodrigue's Formula.

Bessel's Functions, Series Solution, Generating Function, Integral Representation of Bessel's Functions, Hankel Functions, Recurrence Relations, Asymptotic Expansion of Bessel Functions.

References:
1. Simmons, G. F., Differential Equations, (Tata McGraw Hill).
2. Agarwal, Ravi P. and O' Regan D., An Introduction to Ordinary DifferentialEquations, (Springer).
3. Codington, E. A and Levinson, N., Theory of Ordinary Differential Equation, (McGraw Hill).
4. Ince, E. L., Ordinary Differential Equation, (Dover).
5. Piaggio, H. T. H., An Elementary Treatise on Differential Equations andTheir Applications, (G. Bell and Sons, Ltd).
6. Hartman, P., Ordinary Differential Equations, (SIAM).

SEMESTER-II
Course Code: MTM-205
Continuum Mechanics: 50 Marks (4 CP)

Description of motions of a continuum- Material description and spatial description, Material derivative and spatial derivative, Acceleration of a particle in a continuum, Displacement Field, Continuum hypothesis, Continuous media, Body, Configuration, Kinematic equations for rigid bodies.

Theory of deformation and strain: Deformation, Infinitesimal deformation, Infinitesimal strain tensor, Infinitesimal rotation tensor, the rate of change of Material element, the rate of deformation tensor, the spin tensor, the angular velocity vector, equation of conservation of mass, Deformation gradient tensors, Finite strain tensor, Finite strain components in rectangular Cartesian coordinates, Geometrical interpretation of infinitesimal strain components, Strain quadric, Principal strains, Strain invariants, Compatibility equations for linear strains, Rate of strain tensors-its principal values and invariants, Rate of rotation tensor, vorticity vector, velocity gradient tensor, Change of area due to deformation, Change of volume due to deformation.

Theory of stress: Forces in a continuum, Stress tensor, Components of Stress tensor, Principle of moment of momentum, Equations of equilibrium, Symmetry of stress tensor, Shearing and normal stresses, Stress quadric of Cauchy and its properties. Maximum shearing stress, Principal stresses and principal axes of stresses, Invariants of stress tensors, Stress compatibility equations. Equation of motion – Principle of linear momentum, Principle of angular momentum, Equation of motion in cylindrical and spherical coordinates, Rate of Heat Flow into an Element by Conduction, conservation of energy.

Theory of elasticity: Ideal materials, Classical elasticity, Generalized Hooke's Law, Isotropic andanisotropic materials, Constitutive equation for isotropic elastic solid, and anisotropic solid, Strain-energy function, Physical interpretation.

Equations of Hydrostatics, Newtonian Fluid, Incompressible Newtonian Fluid, Navier-Stokes Equation for Incompressible Fluids, Streamline, Path-line, Steady, Unsteady, Laminar and Turbulent Flow, Constitutive equations for Newtonian Fluid, Stress and rate of strain relation.

References:

1. Sokolnikoff I. S., Mathematical Theory of Elasticity. McGraw-Hill
2. Love A. E. H., A Treatise on the Mathematical Theory of Elasticity. Dover Publications Inc.
3. Fung Y. C., Foundations of Solid Mechanics. Prentice Hall
4. Y.C.Fung, A First Course in Continuum Mechanics. Pearson
5. R. N. Chatterjee, Continuum Mechanics.Narosa Publishing House
6. Timoshenko S. and Goodier N, Theory of Elasticity. McGraw Hill Education
7. Ghosh. P. K.,The mathematics of Waves and Vibrations. Macmillan Co. of India

SEMESTER-II
Course Code: MTM-205
Algebraic Topology: 50 Marks (4 CP)

The Fundamental group and covering spaces: Homotopy of paths, Fundamental group, Covering spaces, Fundamental group of the circle, Fundamental group of the punctured plane, Special Van Kampen theorem, Fundamental group of Sn, Seifert - Van Kampen theorem (statement and applications), Fundamental group of surfaces. Essential and inessential maps, Borsuk - Ulam theorem for S2, Fundamental theorem of algebra, Vector fields and fixed points, Brouwer's fixed-point theorem for the disc, Homotopy type, Deformation retract, Strong deformation retract. Jordan separation theorem, Jordan curve theorem (statement only). Classification of covering spaces, General lifting lemma, Existence of coverings, Semilocally simply connectivity, Deck transformations.

Simplicial Homology: Geometric complexes and polyhedra, Orientation of geometric complexes. Chains, Cycles, Boundaries and homology groups, Examples of homology groups, Thestructure of homology groups, Euler - Poincaré theorem, Pseudomanifolds and thehomology groups of Sn. Simplicial approximation, Induced homomorphisms on the homology groups, Brouwerfixed point theorem and related results. Group discussion on some topics related to this course will be conducted. Some assignments will be given on fundamental groups, covering spaces, homotopy type, simplicial homology and other topics of this course.

References:

1. Munkres, J.R., Topology, A First Course, Prentice Hall of India Pvt. Ltd., New Delhi,2000.
2. Croom, F.H., Basic Concepts of Algebraic Topology, Springer, NY, 1978.
3. Bredon, G.E., Topology and Geometry, Springer, India, 2005.
4. Spanier, E.H., Algebraic Topology, McGraw-Hill, 1966.
5. Singer, I.M., Thorpe, J.A., Lecture Notes on Elementary Topology and Geometry,Springer, India, 2003.
6. Hatcher, A., Algebraic Topology, Cambridge University Press, 2003.
7. Dieudonné, J., A History of Algebraic and Differential Topology, 1900 - 1960,Birkhäuser, 1989.

SEMESTER-II
Course Code: MTM-206
Calculus on R^n and Elements of OR: 50 Marks (4 CP)

Unit – 1: Calculus on R^n

Scalar and vector fields, continuity, Directional derivative, total derivative, Jacobian matrix, chain rule, matrix form of the chain rule. Mean value theorem for differentiable functions, sufficient condition for differentiability, sufficient condition for equality of mixed partial derivatives, Taylor's formula for functions from R^n to R^1, Inverse function theorem, Implicit function theorem.

References:

1. T. M. Apostol, Calculus, Volume II, Wiley India Pvt. Limited, 2002.
2. J. R. Munkres, Analysis on Manifolds, Westview Press, 1990.
3. T. Marsden, Basic Multivariate Calculus, Springer, 2013.
4. C. Goffman, Calculus of Several Variables, A Harper International Student reprint, 1965.
5. Michael Spivak, Calculus on Manifolds, Westview Press, 1965.

Unit-II: Elements of Operations Research

Inventory control: Deterministic Inventory control including price breaks and Multi-item with constraints. Queuing Theory: Basic Structures of queuing models, Poisson queues –M/M/1, M/M/C for finite and infinite queue length, Non-Poisson queue -M/G/1. Classical optimization techniques: Single variable optimization, multivariate optimization (with no constraint, with equality constraints).

References:

1. S.D. Sharma, Operations Research: Theory, Method and Application, KedarNath Ram Nath, 2002.
2. F.S. Hillier, Introduction to operations research. Tata McGraw-Hill Education, 2012.
3. S.S. Rao, Engineering optimization: Theory and practice, John Wiley & Sons, 2009.
4. H.A. Taha, Operations research: An introduction. Pearson Education India, 2004.
5. J.K. Sharma, Operations Research: Theory and application, Macmillan Publishers, 2006

SEMESTER-III
Course Code: MTM-301
CBCS – 2: 50 Marks (4 CP)

Set, Venn diagram of sets, algebra of sets, Demorgan’s laws, Binary relation, reflexive relation, symmetric relation, anti-symmetric relation, transitive relation, equivalence relation, partial order relation, poset, mapping, injective mapping, surjective mapping, bijective mapping, permutation, even and odd permutations, Countable sets and cardinality.

Well ordering property, Mathematical induction, Principle of inclusion and exclusion, pigeonhole principle and some of its applications.

Graph, degree of a vertex, walk, path, cycles, subgraph, connected graph, complete graph, bipartite graph, tree and its simple properties.

References:

1. M.K. Sen and B.C. Chakraborty, Introduction to Discrete Mathematics, Books & Allied Ltd.
2. J. G. Chakraborty and P. R. Ghosh, Advanced Higher Algebra (Abstract and Linear), U. N. Dhur and sons private limited
3. S. K. Chakraborty& B. K. Sarkar, Discrete Mathematics, Oxford University Press
4. Swapan Kumar Sarkar, A Textbook of Discrete Mathematics,S Chand & Co Ltd
5. S. K. Mapa, Higher Algebra (Abstract and Linear), Levant Books

SEMESTER-III
Course Code: MTM-302
Numerical Analysis (Theory and Practical in C/R/MATLAB): 50 Marks (4 CP)

Errors and Operators: Inherent error, Round-of error, Truncation error, Approximate numbers, Significant figures, Absolute, relative and percentage errors, Δ, ∇, μ, E, δ operators (definitions and relations among them).

Interpolation and Differentiation: Problems of interpolation, Weierstrass approximation theorem (statement only), Polynomial interpolation, Error of Interpolation, Difference table, Deduction of Newton's forward and backward interpolation formulae with error term, Lagrange's interpolation formula and Divided difference interpolation with error terms, Newton's forward, backward and Lagrange's Differentiation, Various Problems.

Numerical Integration: Open and closed type Newton-Cotes formula, Trapezoidal Rule, Simpson's $1/3^{rd}$rule and their composite forms with geometrical significance, Weddle's rule, Errors and Degree of precision of each method, Gauss's quadrature formula, Various Problems.

Numerical solutions of algebraic and transcendental equations: Location of a real root by tabular method, Bisection method, Secant method, Fixed-point iteration method, Newton-Raphson method (for one and two variables), their geometrical significances and order of convergences, Various Problems.

Numerical solution of ordinary differential equations: Basic ideas, nature of the problems, Picard, Euler, Runge-Kutta (2nd order and 4th order) methods, Various Problems.

Numerical solutions of a system of linear equations: Gauss eliminations method, Gauss-Seidel Method and their convergence, Matrix inversion by Gauss elimination method, L-U Decomposition method.

Eigen value problems: Power method for extreme Eigen values.

Practical (through C/R/MATLAB):
1. Bisection method.
2. Fixed-point iteration method.
3. Newton-Raphson method (for one and two variables).
4. Secant method.
5. Fixed point iteration method
6. Lagrange interpolation.
7. Newton's forward and backward interpolation.
8. Divided difference interpolation.
9. Simpson's $1/3^{rd}$ rule, Trapezoidal Rule and Weddel's rule.
10. Euler's and Runge-Kutta method.
11. Gauss eliminations method.
12. Gauss-Seidel Method.
13. Gauss elimination method.
14. L-U Decomposition method.
15. Power method.

References:

1. Balagurusamy, E. Numerical Methods, Tata McGRAW-Hill
2. Balagurusamy, E. Fundamentals of Computers, Tata McGRAW-Hill
3. Chapra, S.C. and Canale, R.P., Numerical Methods for Engineers, Tata McGRAW-Hill
4. Kanetkar, Y. Let us C, BPB Publications
5. Sastry, S.S., Introductory Methods of Numerical Analysis, Prentice Hall India Pvt. Ltd.
6. Scarborough J.B., Numerical Mathematical Analysis, Oxford & IBH publishing

SEMESTER-III
Course Code: MTM-303
Partial Differential Equation (PDE): 50 Marks (4 CP)

Integral surfaces, Cauchy Problem order equation, Orthogonal surfaces, First order non-linear PDE, Characteristics, Compatible system Charpit's method, Classification and canonical forms of PDE, Classification, reduction to normal form, Solution of equations with constant coefficients by (i) factorization of operators (ii) separation of variables.

Derivation of Laplace and Poisson equation, Boundary Value Problem, Separation of Variables, Dirichlets's Problem and Neumann Problem for a rectangle, Interior and Exterior Dirichlets's problems for a circle, Interior Neumann problem for a circle, Solution of Laplace equation in Cylindrical and spherical coordinates.

Formation and solution of Diffusion equation, Dirac Delta function, Separation of variables method, Solution of Diffusion Equation in Cylindrical and spherical coordinates.

Formation and solution of one-dimensional wave equation, canonical reduction, Initial Value Problem, D'Alembert's solution, Vibrating string, Forced Vibration, Initial Value Problem and Boundary Value Problem for two-dimensional wave equation, Periodic solution of one-dimensional wave equation in cylindrical and spherical coordinate systems, vibration of circular membrane, Uniqueness of the solution for the wave equation, Duhamel's Principle, Examples.

Green's function for Laplace Equation, methods of Images, Eigen function Method, Green's function for the wave and Diffusion equations, Solution of Diffusion and Wave equation by Laplace Transform.

Calculus of Variations: Variation of a functional, Euler-Lagrange equation, Necessary and sufficient conditions for extrema, Variational methods for boundary value problems in ordinary and partial differential equations.

References:

1. Courant and Hilbert, Methods of Mathematical Physics, Vol-II, Wiley-VCH
2. Miller, F. H., Partial Differential Equations, John Wiley & Sons, Inc
3. Petrovsky, I. G., Lectures on Partial Differential Equations, Dover Publications
4. Rao, K. S., Introduction to Partial Differential Equations, Prentice Hall
5. Sneddon, I. N., Elements of Partial Differential Equations, McGraw Hill.
6. Williams, W. E., Partial Differential Equations, Clarendon Press

7. Bernard Dacorogna, Introduction to the Calculus of Variations, Imperial College Press
8. Kung Ching Chang, Lecture Notes on Calculus of Variations, World Scientific

SEMESTER-III
Course Code: MTM-304
Differential Geometry and Manifold Theory: 50 Marks (4 CP)

Unit – I: Differential Geometry
Curves in plane and space, arc-length, reparameterization, closed curves, curvature, Regular surfaces, tangent plane, normalfileds, orientability, first fundamental form, second fundamental form, Normal and Principal curvatures, Gaussian and mean curvatures.

Unit-II: Manifold Theory

Topological manifolds, Differentiable manifolds, smooth maps and diffeomorphisms, curves in a manifold, Tangent Vector, Vector field, integral curve, Lie-bracket, Push-forward Vector field, immersion, submersion, one-parameter group of transformations, Cotangent space, Differential form, wedge product, exterior differentiation.

References:

1. A. Pressley, Elementary Differential Geometry, Springer, 2nd Volume, 2010.
2. Barrett O'Neill, Elementary Differential Geometry, Elsevier, 2006
3. W.M. Boothby, An Introduction to Differentiable Manifolds and Riemannian Geometry, Academic Press, Revised 2003.
4. M. Majumder and Arindam Bhattacharyya, An introduction to smooth Manifold, Springer, 2023.
5. S. Kumaresan, A Course in Differential Geometry and Lie Groups, Hindustan Book Agency.
6. L.W. Tu, An Introduction to Manifolds, Springer, 2007.
7. J.M. Lee, Introduction to Smooth Manifolds, Springer, 2003.
8. S. Lang, Introduction to Differential Manifolds, John Wiley & Sons, New York, 1962.

SEMESTER-III
Course Code: MTM-305
Integral Equations and Integral Transforms: 50 Marks (4 CP)

Integral Equations
Definitions of integral equations and their classifications,Fredholm and Voltera integral equations& their classifications, integral equation of Convolution Type, Eigen value & Eigen function, method of converting an Initial Value Problem (IVP) into a Voltera integral equation, method of converting a Boundary Value Problem (BVP) into a Fredholmintegral equation, homogeneous Fredholm integral equation of the second kind with separable or degenerate kernel, classical Fredholm theory - Fredholm alternative, Fredholm theorem.

Solution of Fredholm and Voltera integral equation of the second kind by successive substitutions & iterative method (Fredholm integral equation only), Reciprocal function, determination of Resolvent kernel and solution of Fredholm integral equation.

Orthonormal system of functions, fundamental properties of Eigen values and function for symmetric kernel, Hilbert theorem, Hilbert-Schmidt theorem.

Integral Transforms

Laplace transform: Definition and basic properties, Laplace integral,Lerch's theorem (statement only), Laplace transforms of elementary functions including Dirac-Delta function and their derivatives,Differentiation and Integration, Convolution, Inverse transform, Applications of Laplace transform.

Fourier transform: Definition and basic properties, Fourier transform of some elementary functions, and their derivatives, Inverse Fourier transform, Convolution theorem and parseval's relationsincluding their applications, Fourier Sine and Cosine transforms, applications of Fourier transform.

Hankel transform: Definition and inversion formula, Hankel transform of derivatives, Finite Hankel transform, Applications of Hankel transform.

References:

1. M. D. Raisinghania, Integral Equations and Boundary Value Problems, S. Chand Publishing
2. R. P. Kanwal, Linear Integral Equations, Birkhauser Boston
3. S. G. Mikhlin, Linear Integral Equations, Hindustan Pub. Corp
4. D. V. Wider, The Laplace Transforms, Dover Publications Inc.
5. P. J. Collins, Differential and Integral Equations, Oxford University Press
6. H. S. Carslaw and J. C. Jaeger, Operational Methods in Applied Mathematics, Dover Publications
7. R. V. Churchill: Operational Mathematics, McGraw-Hill College
8. L. Debnath and D. Bhatta, Integral Transforms and Their Applications, Chapman and Hall/CRC
9. I. N. Sneddon, The Use of Integral Transforms,McGraw-Hill Inc.
10. B. Davies, Integral Transforms and Their Applications, Springer-Verlag New York Inc.

SEMESTER-III
Course Code: MTM-306
(Special Paper-1)
Advanced Algebra-I: 50 Marks (4 CP)

Modules, submodules, quotient module, module homomorphisms, cyclic modules, free modules, finitely generated module over PID. Exact sequences, short and split exact sequences, tensor product of modules, definition and existence, universal mapping property, further properties of the tensor product of modules, projective modules, injective modules, flat Modules, direct sum of projective modules, direct product of injective modules.

Noetherian rings, Artinian rings, Hilbert Basis Theorem, Cohen's Theorem, Primary Decomposition Theorem, Nilradical and Jacobson radical, Nakayama's lemma, prime avoidance, Chinese remainder theorem, extension and contraction of ideals, rings and modules of fractions, local ring, characterizations of local ring, localization, Extended and contracted ideals in rings of fractions,

References:

1. Dummit, D.S. and Foote, R.M., Abstract Algebra, Second Edition, John Wiley & Sons, Inc., 1999.
2. Atiyah, M., MacDonald, I.G., Introduction to Commutative Algebra, Addison-Wesley, 1969.
3. Lang, S., Algebra, Addison-Wesley, 1993.
4. Lam, T.Y., A First Course in Non-Commutative Rings, Springer Verlag.
5. Hungerford, T.W., Algebra, Springer.
6. Jacobson, N., Basic Algebra, II, Hindusthan Publishing Corporation, India.
7. Gopalakrishnan, N. S., Commutative Algebra, 2nd edition, Orient Blackswan, 2015.
8. Balwant Singh, Basic Commutative Algebra, World Scientific Publishing Co Pte Ltd,

SEMESTER-III
Course Code: MTM-306
(Special Paper-1)
Advanced Functional Analysis-I: 50 Marks (4 CP)

Operator Theory: Dual spaces, Representation Theorem for bounded Linear functionals on C[a,b] and L_p spaces, Dual of C[a,b] & L_p spaces, weak & weak*convergence, Reflexive spaces.

Bounded Linear Operators, Uniqueness Theorem, Adjoint of an Operator and its Properties, Normal, Self Adjoint, Unitary, Projection Operators, their Characterizations & Properties. Orthogonal Projections, Characterizations of Orthogonal Projections among all the Projections. Norm of Self Adjoint Operators, Sum & Product of Projections, Invariant Subspaces. Sesquilinearfunctionals on linear spaces and on Hilbert spaces, generalization of Cauchy-Schwarz inequality.

Spectrum of an Operator, Finite Dimensional Spectral Theorem, Spectrum of Compact Operators, Spectral Theorem for Compact Self Adjoint Operators (statement only).

Banach Algebra: Algebra and some properties of the space C(X), Stone-Weierstrass Theorem. Banach Algebra, Banach Sub Algebra, Identity element, invertible elements, existence and representation of the inverse of e-x, resolvent set and resolvent operator, analytic property of the resolvent operators, compactness of spectrum, non-emptiness of the spectrum. Division Algebra, Gelfand-Mazur Theorem, Topological divisors of zero, Spectral radius and its properties, spectral mapping theorem for polynomials, Complex homomorphism, Gleason-Kahane-Zalazko Theorem, Commutative Banach Algebra, Ideals, maximal ideals, Quotient space as a Banach Algebra under certain conditions, Gelfand theory on representation of Banach Algebra, Gelfand transform, weak Topology, weak* Topology, Gelfand Topology, Banach Alaoglu Theorem. Quotient algebra, Banach * algebra, B* algebra, Gelfand Naimark Theorem.

References:

1. W. Rudin, Functional Analysis, Tata McGraw Hill.
2. A. A. Schaffer, Topological Vector Spaces, Springer, 2nd Edn., 1991.
3. G. Bachman & L. Narici, Functional Analysis, Academic Press, 1966.
4. E. Kreyszig, Introductory Functional Analysis with Applications, Wiley Eastern, 1989.
5. Diestal, Applications of Geometry of Banach Spaces.

6. Horvat, Linear Topological spaces.
7. Brown and Page, Elements of Functional Analysis.

SEMESTER-III
Course Code: MTM-306
(Special Paper-1)
Advanced Differential Geometry-I: 50 Marks (4 CP)

Affine connection on manifold, Torsion and curvature tensor fields, Covariant differential, Riemannian connection, Riemannian manifolds, Curvature tensor, Ricci tensor, Einstein manifold, Sectional curvature, Semi-symmetric metric connection, Weyl conformal curvature tensor, different types of curvature tensors, Lie group, Left and Right Translations, Invariant vector fields, Invariant forms, automorphisim, Lie transformation group,

References:

1. W.M. Boothby, An Introduction to Differentiable Manifolds and Riemannian Geometry, Academic Press, Revised 2003.
2. S. Kumaresan, A Course in Differential Geometry and Lie Groups, Hindustan Book Agency.
3. A. Pressley, Elementary Differential Geometry, Springer, 2 nd Volume, 2010.

SEMESTER-III
Course Code: MTM-306
(Special Paper-1)
Mathematical Biology-I: 50 Marks (4 CP)

Mathematical Biology and the modeling process: Introduction, basic steps of Mathematical Modeling, its needs, types of models, limitations.

Single species population models: Malthus growth model, Logistic growth model, Gompertz growth model, single species model with harvesting.

Two species population models: Types of interaction between two species, Lotka-Volterra prey-predator model, Gause model, Lotka-Volterra model of two competing species, Models on mutualism, Gause Model, Kolmogorov Model, Leslie Gower Model.

Epidemic Models**:** Deterministic model of simple epidemic, Infection through vertical and horizontal transmission, General epidemic-Karmac-Mackendric Threshold Theorem, SI, SIR, SIRS models, Basic reproduction number, Stability analysis of SIR and SEIR models.

Discrete system**:** Overview of difference equations, steady state solution and linear stability analysis, Introduction to Discrete Models, Linear Models, Growth models, Decay models, Discrete Prey-Predator model and Epidemic model.

References:

1. H. I. Freedman - Deterministic Mathematical Models in Population Ecology
2. Mark Kot (2001), Elements of Mathematical Ecology, Cambridge Univ. Press.
3. D. Alstod (2001), Basic Populas Models of Ecology, Prentice Hall, Inc., NJ.

4. J.D.Murray (2002), Mathematical Biology, Springer and Verlag.
5. N. T. J. Bailey, The Mathematical Approach to Biology and Medicine.
6. M. Svirezhev and D. O. Logofet, Stability of Biological Communities.

SEMESTER-III
Course Code: MTM-306
(Special Paper-1)
Fluid Mechanics-I: 50 Marks (4 CP)

Lagrange's and Euler's methods in fluid motion, Equations of motion and equation of continuity, Boundary conditions and boundary surface streamlines and paths of particles, Irrotational and rotational flows, velocity potential, Bernoulli's equation, Impulsive action equations of motion and equation of continuity in orthogonal curvilinear coordinate, Euler's momentum theorem and D'Alemberts paradox.

Theory of irrotational motion flow and circulation, Permanence irrotational motion, Connectivity of regions of space, Cyclic constant and acyclic and cyclic motion, Kinetic energy, Kelvin's minimum Energy theorem, Uniqueness theorem, Three Dimensional irrotational motion.

Complex potential, sources, sinks, doublets and their images, Milne-circle theorem, Theorem of Blasius, Motion of circular and elliptic cylinders. Circulation about circular and elliptic cylinder, Steady streaming with circulation, Rotation of elliptic cylinder, Theorem of Kutta and Juokowski, Conformal transformation, Juokowski transformation, Schwartz-Chirstoffel theorem, Motion of a sphere, Stoke's stream function, Source, sinks, doublets and their images with regards to a plane and sphere.

Vortex motion, Vortex line and filament equation of surface formed by streamlines and vortex lines in case of steady motion, Strength of a filament, Velocity field and kinetic energy of a vortex system, Uniqueness theorem rectilinear vortices, Vortex pair, Vortex doublet, Images of a vortex with regards to plane and a circular cylinder, An infinite row of vortices, Two infinite row of parallel vortices, Karman's vortex sheet.

Waves: Surface waves, Paths of particles, Energy of waves, Group velocity, Energy of a long wave.

Reference:
1. Hydrodynamics –A.S. Ramsay (Bell).
2. Hydrodynamics – H. Lamb (Cambridge).
3. Fluid mechanics – L.D. Landau and E.M. Lipschitz (Pergamon), 1959.
4. Theoretical hydrodynamics –L.M. Thomson.
5. Theoretical aerodynamics –I.M. Milne-Thomson; Macmillan, 1958.

SEMESTER-III
Course Code: MTM-306
(Special Paper-1)
Operation Research-I: 50 Marks (4 CP)

(Advanced Optimization Techniques)

Advanced Linear Programming
Revised Simplex Method, Dual Simplex Method, Integer Programming (IP): Concept, differences from LP, Branch and Bound Method, Gomory's Cutting Plane Method, Goal Programming: Formulating multiple-objective problems, priority structures, and weights.
Non-Linear and Dynamic Programming
Non-linear Programming (NLP): Convex and non-convex functions, Local and global optima, Unconstrained Optimization: Steepest descent method. Constrained Optimization: Lagrange multiplier method, Kuhn-Tucker conditions, Penalty function methods, Dynamic Programming: Multistage decision-making, backward recursion, applications in resource allocation and shortest paths.

Network Analysis
Network Analysis: Components of a network, Shortest Path Algorithm – Dijkstra's algorithm, Bellman-Ford algorithm, Loyd-Warshall algorithm, Project Scheduling and PERT/CPM – Construction of project network, Critical Path Method (CPM), Program Evaluation and Review Technique (PERT), Crashing of projects, Time-cost trade-off analysis.

Sensitivity and Post-Optimal Analysis
Concept and importance of sensitivity/post-optimality analysis, Changes in Objective Function Coefficients – Effect of changes in profit/cost coefficients on optimality, Allowable increase and decrease for objective coefficients. Changes in RHS Constraints – Effect of changes in availability of resources, Range of feasibility for RHS values, Addition and deletion of variables and constraints.

References:
1. Operations Research – Kanti Swarup, P.K. Gupta, and Man Mohan (S. Chand & Sons)
2. Operations Research – J.K. Sharma (Macmillan)
3. Operations Research – S. D. Sharma, Kedar Nath
4. Operations Research: Principles and Practice – Ravindran, Phillips, and Solberg (Wiley)
5. Introduction to Operations Research – Frederick S. Hillier and Gerald J. Lieberman (McGraw-Hill)
6. Operations Research: An Introduction – Hamdy A. Taha (Pearson)
7. Linear Programming and Network Flows – Mokhtar S. Bazaraa, John J. Jarvis, Hanif D. Sherali (Wiley)

SEMESTER-III
Course Code: MTM-306
(Special Paper-1)
Plasma Dynamics-I: 50 Marks (4 CP)

Definition of Plasma as an ionized gas, Saha's equation of ionization, Occurrence of plasma in Nature, Plasma as a mixture of different species of charged particles.
First order orbit theory (Single charged particle motion under different force field) (a) Uniform E field, (b) Uniform B field, Larmor orbits and guiding centers, Larmor radius, Larmor

frequency, (c) Uniform E and B fields, Larmor orbits and guiding centers, Larmor radius, Larmor Frequency, (d) The magnetic moment and the magnetization current, (e) Non-uniform B field, (f) Drift velocity, (g) Current due drift velocity, (h) Non-uniform E field, Drift velocity, Current due drift velocity, (i) Motion of charged particle under the action of any force, Drift velocity, Current due drift velocity, (j) Time varying E field, Drift velocity, Current due drift velocity, (k) Time varying B field, Drift velocity, Current due drift velocity.

Elements of kinetic theory (Statistical approach), Single particle phase space, Volume elements, Distribution function, Characterization of plasma with respect to the nature of the distribution function: Homogeneous, Inhomogeneous, Isotropic, Anisotropic.

Derivation of Boltzmann equation, Average values and Macroscopic variables, Derivation of Macroscopic equations (Moment equations): Equation of continuity, Equation of motion, Equation of energy, Assumption on the nature of the distribution function to form a closed and consistent system of macroscopic equations (Equation of State), Cold Plasma limit, The equilibrium state: Maxwellian Distribution, Debye Shielding, The plasma parameter and the criteria for plasma formation.

Plasma-Single fluid approach (MHD): Forces on the charged particles due to the interaction with the electromagnetic field, Basic equations for MHD: Conservation of mass, Conservation of momentum, Conservation of energy, Induction equation, Conservation of magnetic flux, Frozen-in-effect, Alfven theorem, Generalized Ohm’s law, MHD equilibrium.

References:

1. Plasma Physics and controlled Fusion, F.F. Chen, PLENUM PRESS, NEWYORK AND LONDON.
2. Fundamental of Plasma Physics, J. A. Bittencourt, PERGAMON PRESS, NEWYORK AND LONDON.
3. Theory of Plasma waves, T. H. Stix, McGraw Hill.

SEMESTER-III
Course Code: MTM-306
(Special Paper-1)
Solid Mechanics-I: 50 Marks (4 CP)

Analysis of Strain: Curvilinear co-ordinate system, transformation, affine transformation, infinitesimal affine deformation, Strain, geometrical interpretation of strain, Extension, Pure Shear, Simple Shear, Displacement, Displacement in simple extension and simple shear, Homogeneous strain, Relative displacement, Analysis of the relative displacement , Strain corresponding with small displacement , Components of strain, The strain quadric of Cauchy, Principal strain and invariants, Transformation of the components of strain, Types of strain - Uniform extension, Simple extension, Shearing strain, Plane strain, Relations connecting the dilatation, the rotation and the displacement, Resolution of any strain into dilatation and shearing strains, Identical relations between components of strain, Displacement corresponding with given strain, General infinitesimal deformation, Saint-Venant’s equation of compatibility, Finite deformation

Analysis of Stress: Stress tensor, Equation of equilibrium, Stress quadric of Cauchy, Types of stress, (a) Purely normal stress, (b) Simple tension or pressure, (c) Shearing stress, (d) Plane stress, Principal stress and invariants, Maximum normal and share stress, Traction across a plane at a point, Surface tractions and body forces, Equations of motion, Law of equilibrium of surface tractions on small volumes

Equation of elasticity: Work and energy, Existence of the strain-energy-function, Generalized Hooke's law, Homogeneous isotropic media, Elasticity moduli for isotropic media, equilibrium and dynamic equations for an isotropic elastic solid, Strain energy function and its connection with Hooke's law, Elastic constants, Form of the strain-energy-function for isotropic solids, Elastic constants and moduluses of isotropic solids, Moduluses of elasticity, Thermo-elastic equations, Uniqueness of solution, Beltrami- Michell compatibility questions, Saint-Venant's Principle, The equilibrium of an elastic sphere and related problems

Torsion: Torsion of a cylindrical bars, Torsional rigidity, Torsion and Stress functions, Lines of shearing Stress, Method of solution of the torsion problem, Solution of the torsion problem for certain boundaries, Simple problems related to circle, ellipse and equilateral triangle

References:

1. Sokolnikoff I. S., Mathematical Theory of Elasticity.
2. Love A.E. H., A Treastise on the Mathematical Theory of Elasticity.
3. Fung Y.C., Foundations of Solid Mechanics.
4. Y.C.Fung, A First Course in Continuum Mechanics.
5. R.N.Chatterjee, Continuum Mechanics.
6. Timoshenko S. and Goodier N, Theory of Elasticity.
7. Ghosh. P.K., Waves and Vibrations.
8. Prager, N and Hodge , P.G., Theory of Perfectly Plastic Solids.
9. Southwell, R. V., Theory of Elasticity.

SEMESTER-III
Course Code: MTM-306
(Special Paper-1)
Advanced Real Analysis-I: 50 Marks (4 CP)

Ordinal numbers: Order types, well-ordered sets, transfinite induction, ordinal numbers, comparability of ordinal numbers, arithmetic of ordinal numbers, first uncountable ordinal Ω.
Descriptive properties of sets: Perfect sets, decomposition of a closed set in terms of perfect sets of first category, 2nd category and residual sets, characterization of a residual set in a compete metric space, Borel sets of class α, ordinal $\alpha < \Omega$. Density point of a set in R, Lebesgue density theorem.

Functions of some special classes: Borel measurable functions of class α ($\alpha < \Omega$) and its basic properties, comparison of Baire and Borel functions, Darboux functions of Baire class one.

Continuity: The nature of the sets of points of discontinuity of Baire one functions, approximate continuity and its fundamental properties, characterization of approximate continuous functions.

Generalized Integrals: Gauge function, Cousin’s lemma, Role of gauge function in elementary real analysis, Definition of the Henstock integral and its fundamental properties, Reconstruction of primitive function, Cauchy criterion for Henstock integrability, Saks-Henstock Lemma, The Absolute Henstock Integral. The McShane integral. Equivalence of the McShane integral, the absolute Henstock integral and the Lebesgue integral, Monotone and Dominated convergence theorems, The Controlled convergence theorem, Definition and elementary properties of the Perron integral and its equivalence with the Henstock integral.

References:

1. A.M.Bruckner, J.B.Bruckner& B.S.Thomson; Real Analysis; Prentice-Hall, N.Y.1997.
2. I.P.Natanson; Theory of Functions of Real Variable, Vol.I& II; Frederic Ungar Publishing 1955.
3. C.Goffman; Real Functions; Rinehart Company, N.Y, 1953.
4. P.Y.Lee; Lanzhou Lectures on Henstock Integration; World Scientific Press, 1989.
5. J.F. Randolph; Basic Real and Abstract Analysis; Academic Press, N.Y, 1968.
6. S.M.Srivastava; A Course on Borel Sets; Springer, N.Y, 1998.

SEMESTER-III
Course Code: MTM-306
(Special Paper-1)
Advanced Complex Analysis-I: 50 Marks (4 CP)

Analytic Functions: Convex function, Mean values, the function A(r), Borel-Caratheodory’s theorem.
Entire function: Entire functions, entire transcendental functions, order and type of an entire functions, distributions of zeros of analytic functions, the function n(r), Jensen’s theorem and Jensen’s inequality, Order and type in terms of Taylor coefficients, Infinite product of complex number and complex functions, Weierstrass’s factorization theorem, Hadamard’s factorization theorem.

Analytic Continuations: Direct analytic continuations, uniqueness of analytic continuation along a curve, Monodromy theorem and its consequence, the Little Picard Theorem, analytic continuation via Reflection.

Conformal mapping: Normal families, the Riemann mapping Theorem, Schwarz principle of symmetry.

Harmonic Functions: Basic properties of harmonic functions, harmonic function on a disk, subharmonic and superharmonic functions, Dirichlet problems for a disk, Green’s function.

References

1. L. V. Ahlfors; Complex Analysis; McGraw Hill, 1979.
2. R. V. Churchill & J. W. Brown; Complex variables and applications; McGraw Hill.
3. J. B. Conway; Functions of one complex variable; Springer-Verlag, Int. student edition, Narosa Publishing House, 1980.
4. T.W. Gamelin; Complex Analysis; Springer International Edition, 2001.
5. A. S. B. Holland; Theory of entire functions; Academic Press, 1973.

6. S. Lang; Complex Analysis; Forth edition, Springer-Verlag,1999.
7. I. Marcushevich; Theory of functions of a complex variable Vol-I,II,III; Prentice- -Hall,1965.
8. S. Ponnusamy; Foundations of complex analysis; Narosa Publishing House, 1997.
9. H. A. Priestly; Introduction to complex analysis; Clarendon Press, Oxford, 1990.
10. R. Remmert; Theory of Complex Functions; Springer Verlag, 1991.
11. A.R. Shastri; An Introduction to Complex Analysis; Macmilan India, New Delhi, 1999.
12. E. C. Tichmarsh; Theory of functions; Oxford University Press, London, 1939.

SEMESTER-III
Course Code: MTM-306
(Special Paper-1)
Advanced Topology-I: 50 Marks (4 CP)

Nets and Filters: Inadequacy of sequences, Nets & filters, Topology and convergence of nets & filters, Subnets, Ultra nets & Ultra filters, Canonical way of converting nets to filters and vice-versa, Characterizations of compactness and continuity and adherent point in terms of nets and filters, Convergence of nets and filters in product spaces.

Uniformity: Definition and examples, neighbourhoods, bases and subbases of a uniformity, Uniform topology, uniform continuity, product uniformities, uniform isomorphism, relativization and products, Characterization of metrizability, uniformity of pseudometric spaces, uniformity generated by a family of pseudometrics, the gauge of uniformity, Completeness, Cauchy net, Cauchy filter, complete spaces, extension of mappings, completion-existence and uniqueness.

Compactness and uniformity: Diagonal uniformities, uniformity via uniform covers.

Proximity Spaces: Topology induced by a proximity, subspaces and products of proximity spaces, elementary proximity, p-continuity and p-isomorphism, Compactification of proximity spaces-clusters and ultrafilters, Smirnov's theorem.

Ordinal Numbers and Ordinal Spaces: Definition and properties of ordinal numbers, Cardinal numbers vis-`a-vis ordinal numbers, Ordinal spaces, topological properties of ordinal spaces — ω_1 and ω_2 (in particular).

References:

1. J. Dugundji; Topology; Prentice-Hall of India Pvt. Ltd. (1975).
2. R. Engelking; Outline of General Topology; North-Holland Publishing Co., Amsterdam (1968).
3. Ioan James; Topologies and Uniformities; Springer-Verlag (1999).
4. J. L. Kelley; General Topology; D.VanNostrand Co. Inc. (1955).
5. Jun Iti Nagata; Modern General Topology; North-Holland Pub. Amsterdam (1985).
6. S. A. Naimpally& B. D. Warrack; Proximity Spaces; Cambridge University Press (1970).
7. S. Willard; General Topology; Addison-Wesley Publishing Co. (1970).

SEMESTER-III
Course Code: MTM-306
(Special Paper-1)
Harmonic Analysis-I: 50 Marks (4 CP)

Banach Algebra: Normed algebra, Banach algebra, examples of Banach algebra, algebra with involution, C^* -algebra, unitization of Banach algebra, vector-valued analytic functions, resolvent set, resolvent function and its analyticity, spectrum of a point, spectral radius, ideal and maximal ideal of a Gelfand algebra, character space, maximal ideal space with Gelfand topology, Gelfand representation theorem, theory of non-unital Banach algebras.

Topological Group: Basic definition and facts, subgroups, quotient groups, some special locally compact Abelian groups.

Measure Theory on Locally Compact Hausödrff Space: Positive Borel measure, Riesz representation theorem, regularity properties of Borel measures, approximation by continuous functions, Complex measure, absolute continuity of measure, Radon-Nikodym theorem and its consequences, Bounded linear functionals on L^p $(1 \leq p \leq \infty)$, the dual space of $C^0(X)$ for a locally compact Hausdörff space X (the Riesz representation theorem).

Fourier Analysis on Euclidean Spaces: Fourier transform on $L^1(R^n)$ $(n \geq 1)$ and its various properties, inversion of Fourier transform, Fourier transform on $L^2(R^n)$ $(n \geq 1)$, Plancherel theorem.

References:

1. G. B. Folland; A Course in Abstract Harmonic Analysis; CRC Press (1995).
2. Hewitt and Ross; Abstract Harmonic analysis (Vol. I & II); Springer-Verlag (1963).
3. M. Stein and G. Weiss; Introduction to Fourier Analysis on Euclidean Spaces; Princeton University Press (1971).
4. Bachman and Narici; Functional Analysis; Academic Press (1966).
5. C. E. Rickart; General Theory of Banach Algebras; D.VanNostrand Company, Inc.
6. G. F. Simmons; Introduction to Topology and Modern Analysis; McGraw-Hill Book Company (1963).
7. Bachman, Narici and Beckenstein; Fourier and Wavelet Analysis; Springer.
8. Walter Rudin; Real and Complex Analysis; McGraw-Hill Book Company (1921).
9. Walter Rudin; Functional Analysis; Tata McGraw-Hill (1991).
10. R. R. Goldberg; Fourier Transforms; Cambridge, N.Y. (1961).

SEMESTER-IV
Course Code: MTM-401
Probability and Stochastic Process: 50 Marks (4 CP)

Probability:

Random experiments, Simple and compound events, Event space, Classical and frequency definitions of probability and their drawbacks, Axioms of Probability, Statistical regularity, Multiplication rule of probabilities, Baye's theorem, Independent events, Independent random experiments, Independent trials, Bernoulli trials and binomial law, Poisson trials, Random variables, Probability distribution of one dimension, Distribution function, Discrete and continuous distributions – Binomial, Poisson, Gamma, Uniform, Exponential, Beta and Normal distributions.

Transformation of random variables, Two-dimensional probability distributions, Discrete and continuous distributions in two dimensions, Uniform distribution and two-dimensional normal distribution, Conditional distributions, Transformation of random variables in two dimensions.

Mathematical expectation, Mean, variance, moments and central moments, Measures of location, dispersion, skewness and kurtosis, Median, mode, quartiles, Moment generating function, Characteristic function, Two-dimensional expectation, Covariance, Correlation co-efficient, Joint characteristic function, Multiplication rule for mathematical expectations, Conditional expectation, Regression curves, Least square regression lines and parabolas.

Chi-square and t-distributions and their properties (statements only), Tchebycheff's inequality, Convergence in probability, Bernoulli's limit theorem, Law of large numbers. Poisson's approximation to binomial distribution and normal approximation to binomial distribution, Concept of asymptotically normal distribution, Statement of central limit theorem in the case of equal components and the limit theorem for the characteristic functions.

Stochastic Process:
Classification of Random Process, Methods of description of Random Process, Special classes of random processes, Stationarity, SSS process, Representation of random process, Auto-Correlation and cross-correlation function and its processes, Ergodicity.

Gaussian Process and it's properties, Processes depending on stationary Gaussian processes, Poisson Process, probability law and second-order probability function for Poisson process, Mean and auto-correlation of Poisson process, Markov Process, definition of Markov Chain, Chapman-Kolmogorov theorem, Classification of states of Markov chain.

References:

1. Gupta, A. Groundwork of Mathematical Probability and Statistics (Academic Publishers).
2. Gupta, S.C., Kapoor, V.K. Fundamentals of Mathematical Statistics (Sultan Chand and Sons).
3. Johnson, L.J., Kotz, S., Balakrishnan, L. Continuous Univariate Distributions (Vol I and 2) (Wiely and Sons).
4. Johnson, L.J., Kotz, S., Balakrishnan, L. Discrete Multivariate Distributions (Wiely and Sons).
5. Kotz, S., Balakrishnan, L.,Johnson, L.J., Continuous Multivariate Distributions: Models and Applications (Wiely and Sons)
6. Sheldon, R. A first course in probability (Pearson)
7. Veerajan, T. Probability, Statistics and Random Processes (Tata McGraw Hill)

SEMESTER-IV
Course Code: MTM-402
Nonlinear Differential Equation and Dynamical System: 50 Marks (4 CP)

Phase Plane, Paths, and Critical Points, Stable and Unstable Critical Points, Classification of Different Critical points, Critical Points and Paths of Linear Systems, Critical Points and Paths of Nonlinear Systems, Limit Cycles and Periodic Solutions, The Method of Kryloff and Bogoliuboff, Stability, Determination of Stability by Liapunov Method, Orbit of a map, Fixed point, Equilibrium point, Periodic point, Circular map, Configuration Space, Origin of Bifurcation, Hyperbolicity, Quadratic map, Turning point, Hopf Bifurcation, Period Doubling Phenomena, Nonlinear Oscillators, Conservative System, Hamiltonian system, Various types of oscillators in nonlinear system, Solutions of nonlinear differential equations, Poincare map.

References:

1. S. L. Ross, Differential Equations, Wiley, 2018
2. Jordan, D, W and Smith P., Nonlinear Ordinary Differential Equations (2nd Ed.), Oxford University Press, 1987.
3. Sachdev, P.L., Nonlinear Ordinary Differential Equations and Their Applications, Marcel Dekker, Inc., 1991.
4. Blanchard, Devaney, & Hill, Differential Equations, Brooks/Cole, 1998.
5. R. L. Devaney, An introduction to chaotic dynamical system, CRC Press, 2003

SEMESTER-IV
Course Code: MTM-402
Number Theory: 50 Marks (4 CP)

Introduction & Divisibility Theory: Basics, Divisibility, Euclidean Algorithm, Primes and their Distribution, Prime Number Theorem (without proof), Congruence, Linear Congruence and Congruence with prime modulus, Some Diophantine Equations, The Chinese remainder theorem.

Number Theoretic Functions & Applications: Arithmetic functions and the Möbius inversion formula, Greatest Integer Function, Sum of integer squares and Applications.

Fermat’s Theorem & Primitive Roots: Fermat’s little theorem, Euler and Wilsons Theorems, Primitive Roots, Indices, Quadratic Reciprocity, Legendre Symbol, Gauss Theorem.

Applications to Cryptography & Special Topics: Applications to Primality Testing, RSA & cryptography, Fibonacci Numbers, Numbers of Special Form, Continued Fractions and Rational Approximations.

References:

1. Elementary Number Theory by D. Burton, seventh edition, McGraw Hill, 2012.
2. Elementary Number Theory and its Applications by Keneth Rosen, 4th Edition, AddisonWesley, 2000.
3. An Introduction to the Theory of Numbers by I. Niven, H.S. Zuckerman and Hugh L. Montgomery, 5th Edition, Wiley, 1991.

SEMESTER-III
Course Code: MTM-403
Discrete Mathematics: 50 Marks (4 CP)

Counting Technique: Principle of inclusion and exclusion, Pigeon-hole principle, Finite combinatorics, Generating functions, Partitions, Recurrence relations, Linear difference equations with constant coefficients.

Poset and Lattice: Partial and linear orderings, Partially Ordered Set, Covering, Chains and antichains, Lattices, Distributive lattices, Modular Lattices, Boolean Lattices, Complementation.

Graph Theory: Graphs, Undirected and Directed graphs, Basic properties of graphs, Subgraphs, Walk, Path, Cycle, Trail, Euler graphs, Necessary and sufficient condition for a graph to be Euler graph, Konigsberg Bridge Problem, Hamiltonian paths and circuits and their properties, Connected and disconnected graphs, Components of a graph, Complete graph, Complement of a graph, Planar and non-planar graphs, Euler's formula for a planar graph, Non- planarity of the graphs, Basic properties of trees, Spanning tree, BFS and DFS algorithm, Minimal Spanning tree, Kruskal's algorithm, Prim's Algorithm, Rooted tree, Binary search tree.

References:

1. Goodaire, E.G. and Permenter, M. M.Discrete Mathematics (with Graph Theory) (Prentice Hall of India)
2. Rosen, K.H. and Krithivasan,K. Discrete Mathematics and its Applications (Tata McGraw Hill).
3. Sharma, J.K. Discrete Mathematics. (Macmilan)
4. Vasta, B.S. and Vasta, S. Discrete Mathematics (New Age International).
5. Veerajan, V. Discrete Mathematics with Graph Theory and Combinatorics (Tata McGraw Hill)

SEMESTER-IV
Course Code: MTM-404
(Special Paper-2)
Advanced Algebra-II: 50 Marks (4 CP)

Noetherian module, Artinian module, composition series, simple module, semisimple module, Semisimple ring, characterizations of semisimple ring, Wedderburn-Artin theorem on semisimple ring, simple ring, characterization of Artinian simple ring.

Jacobson semisimple ring, relation between Jacobson semisimple ring and semisimple ring, regular ring, idempotents in regular rings, lifting of idempotents, primitive ring, structure of primitive ring, Jacobson-Chevalley density theorem, Wedderburn-Artin theorem on primitive ring.

Prime and semiprime ideal, m-system and n-system, prime radical, prime ring, semiprime ring, ring derivation on prime and semiprime rings, Subdirect sum of rings, representation of a ring as a subdirect sum of rings, subdirectly irreducible ring, Birkhoff theorem on subdirectly irreducible ring, subdirectly irreducible Boolean ring, Stone representation theorem.

Texts / References:
1. T.Y. Lam; Noncommutative Rings; Springer-Verlag.
2. I.N. Herstein; Noncommutative rings; Carus monographs of AMS, 1968.
3. N. Jacobson; Structure of Rings; AMS.
4. L.H. Rowen; Ring theory (student edition); Academic Press, 1991.
5. T.W. Hungerford; Algebra; Springer, 1980.

SEM-IV
Course Code: MTM-404
(Special Paper-2)
Advanced Functional Analysis-II: 50 Marks (4 CP)

Convex sets, convex hull, Representation Theorem for convex hull, Symmetric sets, balanced sets, absorbing sets and their properties, absolutely convex sets, Topological vector spaces, homeomorphisms, local base, locally convex topological vector spaces, bounded sets, totally bounded sets, connectedness and their basic properties, Separation properties of a topological vector space, compact and locally compact topological vector space and its properties on finite dimensional topological vector spaces, convergence of filter, completeness, Frechet space, quotient spaces, separation property by hyperplane on locally convex topological vector spaces, Linear operators over topological vector space, Boundedness and continuity of linear operators, Minkowski functionals and its basic properties, Hyperplanes, Separation of convex sets by Hyperplanes, Extreme points, Krein-Milman Theorem on extreme points, Metrizability of topological vector spaces.

Geometric form of Hahn Banach Theorem, Uniform boundedness principle, open mapping theorem and closed graph theorem for Frechet spaces, Banach-Alaoglu theorem.

Seminorms and its properties, Generating family of seminorms in locally convex topological vector spaces, Criterion for normability of a topological vector space (Kolmogorov Theorem).

Weierstrass Approximation Theorem in C[a,b], best approximation theory in normed linear spaces, uniqueness criterion for best approximation. Separable Hilbert Space, Strict convexity and uniform convexity of a Banach space with examples, Uniform approximation, Haar condition, Haar uniqueness theorem.

Statements of Clarkson’s Renorming Lemma and Milman and Pettit’s theorem, Uniform convexity of a Hilbert space, Reflexivity of a uniformly convex Banch space.

References:
1. W. Rudin, Functional Analysis, TMG Publishing Co. Ltd., New Delhi, 1973**.**
2. E. Kreyszig, Introductory Functional Analysis with Applications, Wiley Eastern, 1989.
3. G. Bachman and L. Narici, Functional Analysis, Academic Press, 1966.
4. A. E. Taylor- Functional Analysis, John Wiley and Sons, New York, 1958.
5. L. Narici& E. Beckenstein, Topological Vector spaces, Marcel Dekker Inc, New York and Basel,1985.
6. A. A. Schaffer, Topological Vector Spaces, Springer, 2nd Edn., 1991.

7. J. Horvoth, Topological Vector spaces and Distributions, Addison-Wesley Publishing Co., 1966.

SEMESTER-IV
Course Code: MTM-404
(Special Paper-2)
Advanced Differential Geometry-II: 50 Marks (4 CP)

Almost Complex manifolds, Nijenhuis tensor, Eigenvalues of the complex structure, Complex manifold, Almost Hermite manifold, Kähler manifold, Holomorphic sectional curvature, Bochner Curvature tensor, Affine connection in Kähler manifold, Conformally flat Kähler manifold. Contact manifold, almost contact manifold, Killing vector field, Properties of φ, K-contact manifold, some curvature properties, Sasakian manifold, φ-sectional curvature, C-Bochner curvature tensor, almost para-contact structure and it's properties.

References:

1. R.S. Mishra, Structure on a Differentiable Manifold and their Applications, ChandramaPrakashani, Allahabad, 1984.
2. K. Yano & M. Kon, Structures on Manifolds, World Scientific, 1984.
3. D.E. Blair, Contact Manifolds in Riemannian Geometry, Lecture note in Math, 509, Springer-Verlag 1976.

SEMESTER-IV
Course Code: MTM-404
(Special Paper-2)
Mathematical Biology-II: 50 Marks (4 CP)

Dynamics of Phytoplankton-Zooplankton system: Introduction, Models on phytoplankton-zooplankton system and its stability, Bio-control in plankton models with nutrient recycling.

Interaction of Ratio-dependent models: Introduction, May's model, ratio dependent models of two interacting species, two prey-one predator system with ratio-dependent predator influence-its stability and persistence.

Continuous models for three or more interacting populations: Food chain models, Stability of food chains, Food chain model with type I response function, and Ratio dependent functional response, Species harvesting in competitive environment, Economic aspects of harvesting in predator-prey systems.

Host-parasite-predator systems, Lotka-Volterra predator-prey model with disease in prey, Lotka-Volterra predator-prey model with disease in predator.

References:

1. Horst R Thieme. Mathematics in Population Biology, Princeton University Press
2. F Brauer and Carlos Castillo-Chavez. Mathematical Models in Population Biology and Epidemiology

3. I. Hanski. Meta Population Ecology, Oxford University Press.
4. Y. M. Svirezhev and D. O. Logofet : Stability of Biological Communities.
5. R. M. May : Stability and Complexity in Model Ecosystem
6. J. D. Murray : Mathematical Biology, Springer and Verlag.
7. F Brauer and Carlos Castillo-Chavez. Mathematical Models in Population Biology and Epidemiology

SEMESTER-IV
Course Code: MTM-404
(Special Paper-2)
Fluid Mechanics-II: 50 Marks (4 CP)

Basic thermodynamics of compressible fluids: Six governing equations of fluid motion, crocco-vazsonyl equation, Propagation of small disturbances in a gas, Mach number.

Dynamics similarity of two flows, Circulation theorem, Permanence of irrotational motion, Bernoulli's integral for steady isentropic and irrotational motion, Polytropic gas, Critical speed, Equation satisfied velocity potential and stream functions, Prandtl-Mayer fluid past a convex corner.

Steady flow through a De Level nozzle, Normal and oblique shock wave shock polar diagramone dimensional similarity flow, Steady linearized subsonic and supersonic flows, Prandtl-Glauert transformation, thin supersonic wind Ackeret's formula.

Legendre and molenbroek transformations, Chaplygin's equation for stream function, Solution of Chaplygin's equation, Subsonic gas jet problem limiting line.

References:

1. Hydrodynamics –A.S. Ramsay (Bell).
2. Hydrodynamics – H. Lamb (Cambridge).
3. Fluid mechanics – L.D. Landau and E.M. Lipschitz (Pergamon), 1959.
4. Theoretical hydrodynamics –L.M. Thomson.
5. Theoretical aerodynamics –I.M. Milne-Thomson; Macmillan, 1958.
6. Introduction to the theory of compressible flow –Shih-I. Pai; Van Nostrand, 1959.
7. Inviscid gas dynamics – P. Niyogi, Mcmillan, 1975(India)
8. Gas dynamics – K. Oswatitsch (English tr.) academic press, 1956

SEMESTER-IV
Course Code: MTM-404
(Special Paper-2)
Operation Research-II: 50 Marks (4 CP)
(Stochastic Models and Decision Theory)

Queuing Theory: Elements of a queuing system: Arrival and service processes, Kendall notation. M/M/1 queue: Derivation of performance measures (waiting time, queue length). M/M/c queue: Multi-server model, steady-state analysis. Some special models (M/G/1). Applications

Inventory Models: Basic EOQ models (with and without shortages), production lot size models. Price breaksand quantity discounts. Probabilistic inventory models: Demand as a random variable, reorder point systems. Safety stock: Service level, fill rate, stock-out risks. Inventory underlead time and review periods.

Stochastic Processes: Definition and classification of stochastic process. Markov Chains: Definition, transition probability matrices, Chapman-Kolmogorov equations, Classification of states: transient, recurrent, periodic, ergodic, Limiting and stationary distributions, Poisson process and itsextensions, birth and death process. Renewal processes in continuous time, renewal equation; Stopping time, Wald's equation, renewal theorems, residual and excess lifetime, renewal reward processes.

Simulation and Monte Carlo Methods: Introduction to Simulation: Types, advantages and limitations. Random number generation: Congruential methods, tests for randomness. Monte Carlo simulation: Conceptand methodology. Simulation of systems: Queues (single and multi-channel), inventory systems. Use of simulation software: Arena, Python.

References:

1. Operations Research - S. D. Sharma, KedarNath
2. Operations Research: An Introduction – Hamdy A. Taha (Pearson)
3. Introduction to Probability Models – Sheldon M. Ross (Academic Press)
4. An Introduction to Decision Theory – Martin Peterson (Cambridge University Press)
5. Quantitative Techniques in Management – N.D. Vohra (Tata McGraw Hill)
6. Simulation Modeling and Analysis – Averill M. Law and W. David Kelton (McGraw-Hill)

SEMESTER-IV
Course Code: MTM-404
(Special Paper-2)
Plasma Dynamics-II: 50 Marks (4 CP)

Vlasov – Boltzmann self-consistent equations in collision less plasma, Plasma oscillations, Landau damping.

Waves in warm field free plasma, Dielectric tensor and general dispersion relation – High frequency approximation & Low frequency approximation.

Waves in cold homogeneous magneto plasma, Dielectric Tensor and general dispersion relation, Cut – Off & Resonance, Group velocity, Wave normal surface, Refractive Index, Refractive index surface, Different types of wave modes for different types of approximation.

Waves in warm homogeneous magneto plasma, Dielectric tensor and general dispersion relation, Two fluid model, Single fluid model, Two stream instability. Characterization of different waves in plasmas.

Electron waves (electrostatic): Plasma oscillations, Ion acoustic wave in a collisionless unmagnetized Plasma, Ion acoustic wave in a collisionless magnetized Plasma (a) propagating along the direction of the magnetic field (b) propagating obliquely to the direction of the magnetic field, Alfven wave, Magnetosonic wave.

Nonlinear wave processes in plasma, Derivation of KdV-ZK equation for ion-acoustic wave & Alfven wave and their soliton solution.

References:

1. Plasma Physics and controlled Fusion, F.F. Chen, PLENUM PRESS, NEWYORK AND LONDON.
2. Fundamental of Plasma Physics, J. A. Bittencourt, PERGAMON PRESS, NEWYORK AND LONDON.
3. Theory of Plasma waves, T. H. Stix, McGraw Hill.

SEMESTER-IV
Course Code: MTM-404
(Special Paper-2)
Solid Mechanics-II: 50 Marks (4 CP)

Two dimensional problem: Plane stress, Generalized plane stress, Airy stress function, General solution of Biharmonic equation, Stresses and displacements in terms of complex potentials, Simple problems, Stress function appropriate to problems of plane stress, Problems of semi-infinite solids with displacements or stresses prescribed on the plane boundary

Waves: Propagation of waves in an isotropic elastic solid medium, Waves of dilatation and waves of distortion, Motion of a surface of discontinuity- Kinematical conditions, Motion of a surface of discontinuity- Dynamical conditions, Velocity of waves in isotropic medium, Velocity of waves in aeolotropic solid medium, Plane waves, Elastic surface waves such as Rayleigh and Love waves

Variational methods: Theorems of minimum potential energy, Theorems of minimum complementary energy, Reciprocal theorem of Betti and Rayleigh, Deflection of elastic string, Central line of a beam elastic membrane, Torsion of cylinders, Variational problem related to biharmonic equation, Solution of Euler's equation by Ritz, Galerkin and Kantorovich methods

References

1. Sokolnikoff I. S. : Mathematical Theory of Elasticity.
2. Love A.E. H. : A Treastise on the Mathematical Theory of Elasticity.
3. Fung Y.C. : Foundations of Solid Mechanics.
4. Y.C.Fung- A First Course in Continuum Mechanics.
5. R.N.Chatterjee – Continuum Mechanics.

6. Timoshenko S. and GoodierN : Theory of Elasticity.
7. Ghosh. P.K. : Waves and Vibrations.
8. Prager, N and Hodge , P.G. : Theory of Perfectly Plastic Solids.
9. Southwell, R. V. : Theory of Elasticity.

SEMESTER-IV
Course Code: MTM-404
(Special Paper-2)
Advanced Real Analysis-II: 50 Marks (4 CP)

Derivative: Banach-Zarecki theorem, derivative and integrability of absolutely continuous functions, Lebesgue point of a function, determining a function by its derivative.

General Measure and Integration: Additive set functions, measure and signed measures, limit theorems, Jordan and Hahn decomposition theorems, complete measures, integrals of non-negative functions, integrable functions, absolute continuous and singular measures, Radon-Nikodym theorem, Radon-Nikodym derivative in a measure space.

Fourier Series: Fourier series of functions of class L, Fejer-Lebesgue theorem, integration of Fourier series, Cantor-Lebesgue theorem on trigonometric series, Riemann's theorem on trigonometric series, uniqueness of trigonometric series.

Distribution Theory: Test functions, compact support functions, distributions, operation on distributions, local properties of distributions, convergence of distributions, differentiation of distributions and some examples, derivative of locally integrable functions, distribution of compact support, direct product of distributions and its properties, convolution and properties of convolutions.

References:

1. A.M.Bruckner, J.B.Bruckner&amp,B.S.Thomson, Real Analysis, Prentice-Hall, N.Y.1997.
2. R.L.Jeffery, The Theory of Functions of a Real Variable, Toronto University Press,1953.
3. I.P.Natanson, Theory of Functions of Real Variable, Vol.I& II, Frederic Ungar Publishing 1955.
4. F.G.Friedlander, Introduction to the Theory of Distributions, Cambridge Univ Press,1982
5. H.L.Royden, Real Analysis, Macmillan, N.Y, 1963.
6. S. Kesavan, Topics in Functional Analysis and its Applications, Wiley Eastern Ltd, New Delhi, 1989.

SEMESTER-IV
Course Code: MTM-404
(Special Paper-2)
Advanced Complex Analysis-II: 50 Marks (4 CP)

Meromorphic Functions: Some basic properties of meromorphic functions, Mittag-Leffler's theorem, application of Mittag-Leffler's theorem for simple poles, Gamma functions and its properties, Riemann zeta functions, Riemann's functional equations, Runge's theorem.

The Range of Analytic Functions: Bloch's theorem, The little Picard theorem, Schottky's theorem, The Great Picard theorem.

Inverse and Implicit Functions of Complex Variables: Inverse functions — the single valued case, the multivalued case, examples of Lagrange's series, functions of two variables, Weierstrass Preparation theorem, the Implicit function theorem.

Univalent Functions: Basic properties of univalent functions, sequence of univalent functions, necessary and sufficient conditions for a function to be univalent, Area theorem, Distortions theorem.

References

1. L. V. Ahlfors, Complex Analysis, McGraw Hill, 1979.
2. R. V. Churchill & amp, J. W. Brown, Complex variables and applications, McGraw Hill.
3. J. B. Conway, Functions of one complex variable, Springer-Verlag, Int. student edition, NarosaPublishing House, 1980.
4. T.W. Gamelin, Complex Analysis, Springer International Edition, 2001.
5. A. S. B. Holland, Theory of entire functions, Academic Press, 1973.
6. S. Lang, Complex Analysis, Forth edition, Springer-Verlag,1999.
7. I. Marcushevich, Theory of functions of a complex variable Vol-I, Vol-II, Vol-III, Prentice- -Hall,1965.
8. S. Ponnusamy, Foundations of complex analysis,Narosa Publishing House, 1997.
9. H. A. Priestly, Introduction to complex analysis, Clarendon Press, Oxford, 1990.
10. R. Remmert, Theory of Complex Functions, Springer Verlag, 1991.
11. A.R. Shastri, An Introduction to Complex Analysis,Macmilan India, New Delhi, 1999.
12. E. C. Tichmarsh, Theory of functions, Oxford University Press, London, 1939.

SEMESTER-IV
Course Code: MTM-404
(Special Paper-2)
Advanced Topology-II: 50 Marks (4 CP)

Paracompactness: Different types of refinements and their relationships, paracompactness in terms of open locally finite refinements, Michael's theorem, fully normal spaces, Stone's coincidence theorem, paracompactness in terms of open delta refinements, cushioned refinements etc. A. H. Stone's theorem concerning paracompactness of metric spaces, partition of unity, properties of paracompact spaces with regard to subspaces, product etc.

Function Space: Pointwise convergence topology and uniformity, compact-open topology, uniqueness of jointly continuous topology, uniform convergence on a family of sets, completeness, uniform convergence on compacta, K-spaces, compactness and equi-continuity. The Ascoli theorem, Even continuity, topological Ascoli theorem, basis for Z^Y, compact subsets of Z^Y, sequential convergence in the c-topology, metric topologies — relation to the C-topology, pointwise convergence, comparison of topologies in Z^Y. The spaces C(Y) — continuity of the algebraic operations, algebras in $\hat{C}(Y, C)$, Stone-Weierstrass theorem, the metric space C(Y), embedding of Y in C(Y), The ring $\hat{C}(Y)$.

Metrization: Metrization theorems of Nagata–Smirnov, Bing, Smirnov, A.H.Stone, Arhangeliskii etc.
Elements of Dimension Theory: Menger-Urysohn dimension (the small inductive dimension) of aspace, indX and IndX, dim X, associated results, specially in connection with 0-dimensional or totally disconnected spaces and βX etc.

References:

1. J. Dugundji, Topology, Prentice-Hall of India Pvt. Ltd. (1975).
2. R. Engelking, Outline of General Topology, North-Holland Publishing Co, Amsterdam. (1968).
3. J. L. Kelley, General Topology,D.VanNostrand Co. Inc. (1955).
4. Jun Iti Nagata, Modern General Topology, North-Holland Pub. Amsterdam (1985).
5. S.Willard, General Topology, Addison-Wesley Publishing Co. (1970).
6. W. Hurewicz and H. Wallman, Dimension Theory; Princeton University Press (1948).

SEMESTER-IV
Course Code: MTM-404
(Special Paper-2)
Harmonic Analysis-II: 50 Marks (4 CP)

Haar measure and Haar integral: Invariant measure and Integration, existence and uniqueness of Haar measure and Haar integral on locally compact topological group, Examples of Haar measures, Haar Integration.

Basic Representation Theory: Unitary representations, Schur's lemma, representations of a group andits group algebra, Gelfand-Raikov theorem.

Fourier Analysis on Locally Compact Abelian Group: The dual group, the Fourier transform, Fourier-Stieltjes transforms, positive-definite functions, Bochner's theorem, the inversion theorem, the Plancherel theorem, Pontryagin duality theorem, representations of locally compact Abelian groups, closed ideals in L 1 (G) for a locally compact Abelian group G, Wiener's Tauberian theorem.

References:

1. Hewitt and Ross, Abstract Harmonic analysis (Vol. I &amp, II), Springer-Verlag (1963).
2. Walter Rudin, Fourier Analysis on Groups, Interscience Publishers (1962).
3. G. B. Folland, A Course in Abstract Harmonic Analysis, CRC Press (1995).
4. Bachman and Narici, Elements of Abstract Harmonic Analysis, Academic Press, New York (1964).
5. L. H. Loomis, An Introduction to Abstract Harmonic Analysis, D.VanNostrand Company Inc. (1953).
6. Y. Katznelson, An Introduction to Harmonic Analysis, Dover Publications, Inc. (1976).